Abstract

If we consider the contact process with infection rate λ on a random graph on n vertices with power law degree distributions, mean field calculations suggest that the critical value λ_c of the infection rate is positive if the power $\alpha>3$. Physicists seem to regard this as an established fact, since the result has recently been generalized to bipartite graphs by Gómez-Gardeñes et al. [*Proc. Natl. Acad. Sci. USA* **105** (2008) 1399–1404]. Here, we show that the critical value λ_c is zero for any value of $\alpha>3$, and the contact process starting from all vertices infected, with a probability tending to 1 as $n\to\infty$, maintains a positive density of infected sites for time at least $\exp(n^{1-\delta})$ for any $\delta>0$. Using the last result, together with the contact process duality, we can establish the existence of a quasi-stationary distribution in which a randomly chosen vertex is occupied with probability $\rho(\lambda)$. It is expected that $\rho(\lambda)\sim C\lambda^{\beta}$ as $\lambda\to 0$. Here we show that $\alpha-1\le\beta\le 2\alpha-3$, and so $\beta>2$ for $\alpha>3$. Thus even though the graph is locally tree-like, β does not take the mean field critical value $\beta=1$.